



*Math in Focus: Singapore Math* National Institute  
July 16-17 2013 | Philadelphia PA

# *Computational Fluency*

# Just the Facts...

- **Is it a necessity?**
- **What do we expect?**
- **What are the roadblocks?**

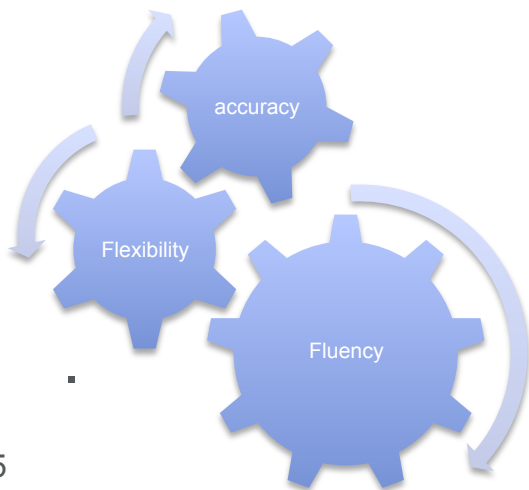
Try These...

$$22 \times 12 =$$

What do the writers of *Math in Focus-Singapore Math* believe?

*What evidence do you have?*

*Remembering and  
memorizing are not the  
same...*

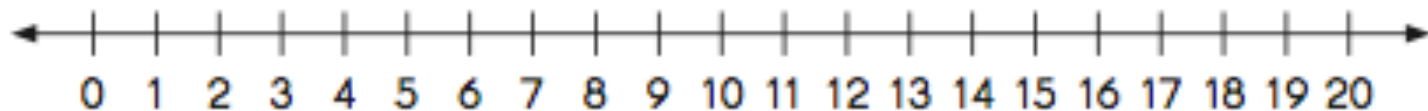


- Conceptual understanding & the ability to visualize.
- Strategies that strengthen understanding and promote efficiency.
- Fluency from understanding

At the aquarium, Hector sees 3 octopuses.  
Each octopus has 8 tentacles.  
How many tentacles do the octopuses have in all?

$$2 \times 4 \times 2 = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$





**Fill in the missing numbers.**

**3.** 8 sixes =  $8 \times$  \_\_\_\_\_

**4.**  $5 \times 6 =$  \_\_\_\_\_ sixes

**5.**  $7 + 7 + 7 + 7 + 7 + 7 = 6 \times$  \_\_\_\_\_

**6.**  $6 + 6 + 6 + 6 + 6 + 6 + 6 = 7 \times$  \_\_\_\_\_

**7.**  $10 \times 6 = 6 \times$  \_\_\_\_\_

**8.** 3 sixes =  $6 + 6 +$  \_\_\_\_\_

$$5 \times 6 = \underline{\hspace{2cm}}$$

$$7 \times 6 = \underline{\hspace{2cm}} \text{ groups of } 6$$

$$= 5 \text{ groups of } 6 + \underline{\hspace{2cm}} \text{ groups of } 6$$

$$= \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

# Be Systematic:

Grade	Standard	Required Fluency
Kindergarten	K.OA.5	Add within 5
		Subtract within 5
First Grade	1.OA.6	Add within 10
		Subtract within 10
Second Grade	2.OA.2	Add within 20
		Subtract within 20
	2.NBT.5	Add within 100
		Subtract within 100
Third Grade	3.OA.7	Multiply within 100
		Divide within 100
	3.NBT.2	Add within 1,000
		Subtract within 1,000
Fourth Grade	4.NBT.4	Add within 1,000,000
		Subtract within 1,000,000
Fifth Grade	5.NBT.5	Multi-digit multiplication





# 4 Addition Strategies

When zero is added to a number, the result is the number itself.

Know the doubles, use them to figure out the near doubles.

Put the greater number in your head, then count on the lesser number.

Determine how many more to make ten, add the additional.



## Zero in Addition

+	0	1	2	3	4	5	6	7	8	9
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9
1	1+0									
2	2+0									
3	3+0									
4	4+0									
5	5+0									
6	6+0									
7	7+0									
8	8+0									
9	9+0									

When zero is added to a number, the result is the number itself.



Put the greater number in your head, then count on the lesser number.

$8 + 2 \dots 8, 9, 10$

$3 + 9 \dots 9, 10, 11, 12$

### Counting on 1, 2, 3

+	0	1	2	3	4	5	6	7	8	9
0										
1		1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9
2		2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9
3		3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9
4		4+1	4+2	4+3						
5		5+1	5+2	5+3						
6		6+1	6+2	6+3						
7		7+1	7+2	7+3						
8		8+1	8+2	8+3						
9		9+1	9+2	9+3						

## Doubles, Near Doubles

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4					4+4	4+5	4+6			
5					5+4	5+5	5+6	5+7		
6					6+4	6+5	6+6	6+7	6+8	
7						7+5	7+6	7+7	7+8	7+9
8							8+6	8+7	8+8	8+9
9								9+7	9+8	9+9

Memorize the doubles, use them to figure out the near doubles.

$$7 + 7 = 14$$

7 + 6 = one less than 14



## Making a Ten

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4								4+7	4+8	4+9
5									5+8	5+9
6										6+9
7					7+4					
8					8+4	8+5				
9					9+4	9+5	9+6			

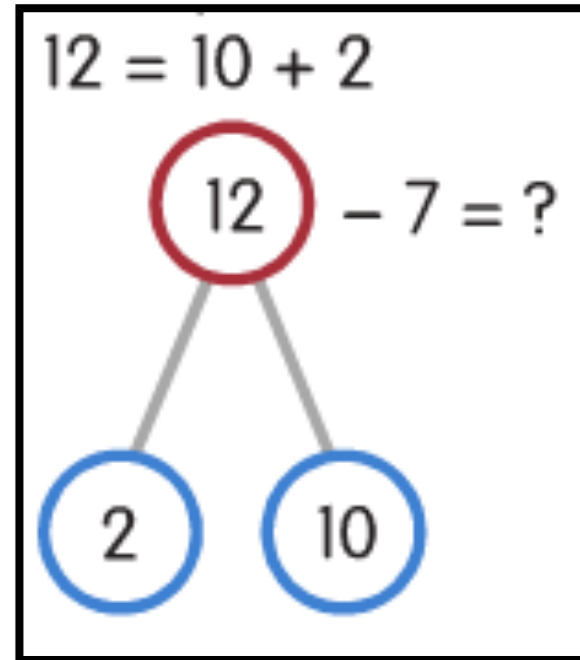
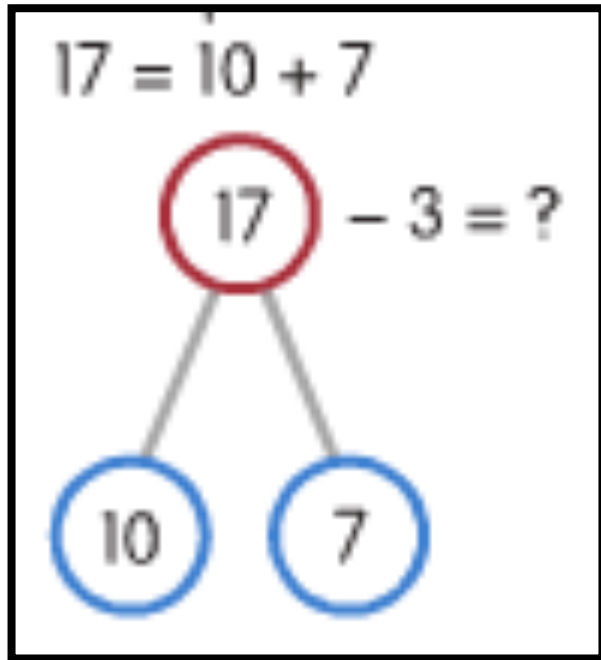
Determine how many more to make ten, add the additional.  
 $4 + 8 = 8 + 2 + 2$   
2 more to ten and 2 more makes 12

3					3+4	3+2	3+1			
8					8+4	8+2				
1					1+4					





# Putting the Strategies into Practice





# 4

## Subtraction Strategies

When you subtract zero from any number, the result is the number itself.

Start with the lesser number, count up to the greater.

Use doubles, near doubles and making tens.

Put the greater number in your head, count back the number of places indicated by the lesser number.



## Subtracting with Zero

-	0																			
0	0-0	1																		
1	1-0	1-1	2																	
2	2-0		2-2	3																
3	3-0			3-3	4															
4	4-0				4-4	5														
5	5-0					5-5	6													
6	6-0						6-6	7												
7	7-0							7-7	8											
8	8-0								8-8	9										
9	9-0									9-9										
	10																			
		11																		
			12																	
				13																
					14															
						15														
							16													
								17												
									18											
										19										

When you subtract zero from any number, the result is the number itself.





## Counting Up 1, 2, 3

-	0																		
0		1																	
1			2																
2				3															
3					4														
4						5													
5						5-4													
6						6-4	6-5												
7						7-4	7-5	7-6											
8							8-5	8-6	8-7										
9								9-6	9-7	9-8									
	10								10-7	10-8	10-9								
		11								11-8	11-9								
			12								12-9								
				13															
					14														
						15													
							16												
								17											
									18										
										19									

Start with the lesser number, count up to the greater. Keep track of how many places you count up.



## Using Addition Facts You Know

Use doubles, near doubles and making tens.

-	0																		
0		1																	
1			2																
2				3															
3					4														
4						5													
5							6												
6								7											
7									8										
8										8-4									9
9										9-4	9-5								
	10									10-4	10-5	10-6							
		11								11-4	11-5	11-6	11-7						
			12							12-4	12-5	12-6	12-7	12-8					
				13						13-4	13-5	13-6	13-7	13-8	13-9				
					14					14-5	14-6	14-7	14-8	14-9					
						15				15-6	15-7	15-8	15-9						
							16			16-7	16-8	16-9							
								17		17-8	17-9								
									18	18-9									
																			19

= Doubles, Near Doubles  
 = Making a Ten

= Making a Ten

= Doubles, Near Doubles





# 4

## Multiplication Strategies

*Relate the doubles to learning the 2's, and use doubling the doubles to find the 4's.*

*- Begin by knowing the 2's 5's and 10's. Add a row or subtract a row.*

*- Use the associative property.*

*-Use what you know...Use the 2's and 5's to find the 7's. Use the 2's and 10's to find the 8's.*



Skip count by 3's or 5's to find these facts.

### Skip Counting (3's, 5's)

x	0	1	2	3	4	5	6	7	8	9
0										
1										
2						2x5				
3				3x3	3x4	3x5	3x6	3x7	3x8	3x9
4				4x3		4x5				
5			5x2	5x3	5x4	5x5	5x6	5x7	5x8	5x9
6				6x3		6x5				
7				7x3		7x5				
8				8x3		8x5				
9				9x3		9x5				



Relate the doubles to learning the 2's, and use doubling the doubles to find the 4's.

<b>Doubles (2's, 4's)</b>										
x	0	1	2	3	4	5	6	7	8	9
0										
1										
2			2x2	2x3	2x4		2x6	2x7	2x8	2x9
3			3x2							
4			4x2		4x4		4x6	4x7	4x8	4x9
5										
6			6x2		6x4					
7			7x2		7x4					
8			8x2		8x4					
9			9x2		9x4					

## Using Tens (9's)

x	0	1	2	3	4	5	6	7	8	9
0										
1										
2										2 x 9
3										3 x 9
4										4 x 9
5										5 x 9
6										6 x 9
7										7 x 9
8										8 x 9
9			9 x 2	9 x 3	9 x 4	9 x 5	9 x 6	9 x 7	9 x 8	9 x 9

Multiply by ten then subtract one group  
 $9 \times 7 = (10 \times 7) - 7$ .

9			9 x 2	9 x 3	9 x 4	9 x 5	9 x 6	9 x 7	9 x 8	9 x 9
8										
7										



## Use What You Know (6's, 7's, 8's)

x	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

6x6	6x7	6x8	6x9
7x6	7x7	7x8	7x9
8x6	8x7	8x8	8x9
9x6	9x7	9x8	9x9

Use the 2's and 5's to find the 7's. Use the 2's and 10's to find the 8's.

6										
7										
8										

6x6	6x7	6x8	6x9
7x6	7x7	7x8	7x9
8x6	8x7	8x8	8x9
9x6	9x7	9x8	9x9



# Know what they know...

Find all possible values of P, Q, R, and S.  
Regrouping is not used in the problem.  
P, Q, R, and S represent different numbers.

$$\begin{array}{r} \phantom{\times} P 3 Q \\ \times \phantom{P} \phantom{3} 3 \\ \hline R 9 S \end{array}$$

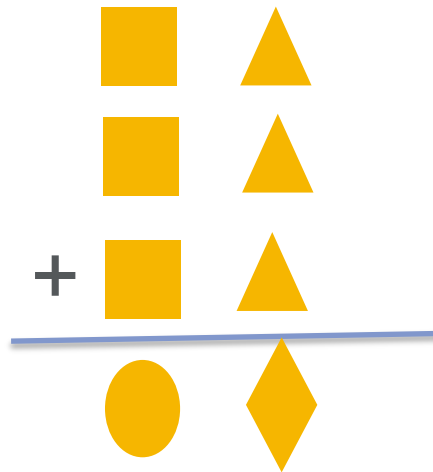
# Invest in thinking...

$$456 = \underline{\hspace{2cm}} + 19$$

What is the missing number?

$$\begin{array}{r} \square \quad 8 \quad \square \\ - \quad 4 \quad \square \quad 9 \\ \hline 3 \quad 6 \quad 7 \end{array}$$

# Invest in thinking...





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*Thank You*